Prescription

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ABSTRACT

We obtain the energy distribution of the gamma metric using the energy-momentum complex of Møller. The result is the same as obtained by Virbhadra in the Weinberg prescription.

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1 INTRODUCTION

Energy-momentum is regarded as the most fundamental conserved quantity in physics, and associated with a symmetry of space-time geometry. According to Noether's theorem and translations invariance, one could define a conserved energy-momentum $T^{\mu\nu}$ as a consequence of its satisfying the differential conservation law $\partial_\nu T^{\mu\nu} = 0$. However, in a curve space-time where the gravitational field is presented, the differential conservation law becomes

$$\nabla_\nu T^{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}(\sqrt{-g}T^{\mu\nu}) - \frac{1}{2}g^{\nu\rho}\frac{\partial g^{\nu\rho}}{\partial x^\lambda}T^{\mu\lambda} = 0, \tag{1}$$

and generally does not lead to any conserved quantity. Early energy-momentum investigations attempted to determine the conserved energy-momentum for the gravitational field and the matter located in it, and led to energy-momentum complex

$$\Theta^{\mu\nu} = \sqrt{-g}(T^{\mu\nu} + t^{\mu\nu}), \tag{2}$$

which satisfies the differential conservation equation $\partial_\nu \Theta^{\mu\nu} = 0$. Here, $T^{\mu\nu}$ is the energy-momentum tensor of matter and $t^{\mu\nu}$ is regarded as the contribution of energy-momentum from the gravitational field. There are various energy-momentum complexes, including those of Einstein [1], Tolman [2], Papapetrou [3], Bergmann [4], Landau and Lifshitz [5], Møller [6], and Weinberg [7]. On the other way, a different idea, quasilocal (i.e., associated with a closed 2-surface) was proposed. The Hamiltonian for a finite region,

$$H(N) = \int_\Sigma N^\mu \mathcal{H}_\mu + \oint_{S=\partial\Sigma} \mathcal{B}(N), \tag{3}$$

generates the space-time displacement of a finite spacelike hypersurface Σ along a vector field N^μ. Noether's theorem guarantee that $\mathcal{H}_\mu$ is proportional to the filed equation. Consequently, the value depends only on the boundary term $\mathcal{B}$, which gives the quasilocal energy-momentum. Moreover, there are also a large number of definitions of quasilocal mass [8, 9]. In their recent article, Chang *et al.* [9] showed that every energy-momentum complex can be associated with a particular Hamiltonian boundary term. So the energy-momentum complexes may also be considered as quasilocal.

Though Penrose [10] points out that a quasilocal mass is conceptually important. However, Bergqvist [11] studied several different definitions of quasilocal masses for the Reissner- Nordström and Kerr space-times and came to the conclusion that not even two of these definitions gave the same results. On the contrary, several energy-momentum complexes have been showing a high degree of consistency in giving the same energy distribution for a given space-time. Recently, Virbhadra and his collaborators [12, 13, 14, 15, 16] have investigated that for a given space-time (like as the Kerr- Newman, the Vaidya, the Einstein-Rosen, the Bonnor-Vaidya and all Kerr-Schild class space-time) different energy-momentum complexes (the Einstein, the Landau-Lifshitz, the Papapetrou, the Tolman, The Weinberg, etc.) give the same energy distribution. Moreover some interesting results [12, 17, 18, 19, 20] led to the conclusion that in a given space-time (the Reissner-Nordström, the Kerr-Newman, the Garfinkle-Horowitz-Strominger, the de Sitter-Schwarzschild, and the charged regular metric, etc.) the energy distribution according to the energy-momentum complex of Møller is different from of Einstein. But in some specific case [6, 17] (the Schwarzschild, the Janis-Newman-Winicour metric, etc.) there are the same. Recently, the energy distribution in the Weinberg prescription obtained by Virbhadra [21] using the gamma metric, is given as

$$E = m\,\gamma. \tag{4}$$

So, in this letter, we evaluate the energy distribution of the gamma metric by using Møller energy-momentum complex, and compare with the result obtained by Virbhadra with Weinberg energy-momentum complex.

2 ENERGY IN THE MØLLER PRESCRIPTION

First, the well-known gamma metric [21, 22], a static and asymptotically flat exact solution of Einstein vacuum equations, is given as

$$ds^2 = (1-\frac{2m}{r})^{\gamma}dt^2-(1-\frac{2m}{r})^{-\gamma}\left[\left(\frac{\Delta}{\Sigma}\right)^{\gamma^2-1}dr^2+\frac{\Delta^{\gamma^2}}{\Sigma^{\gamma^2-1}}d\theta^2+\Delta sin^2\theta d\phi^2\right], \tag{5}$$

where

$$\begin{aligned} \Delta &= r^2-2mr, \\ \Sigma &= r^2-2mr+m^2sin^2\theta. \end{aligned} \tag{6}$$

For $|\gamma| = 1$ the metric is spherically symmetric and for $|\gamma| \neq 1$, it is axially symmetric. In the situation $|\gamma| = 1$, the gamma metric reduces to the Schwarzschild space-time. However, in another situation $|\gamma| \neq 1$, the gamma metric gives the Schwarzschild space-time with negative mass, as putting $m = -M(M > 0)$ and carrying out a coordinate transformation $r \to R = r + 2M$ one gets the Schwarzschild space-time with positive mass.

Next, let us consider the Møller energy-momentum complex which is given by [6]

$$\Theta_\nu{}^\mu = \frac{1}{8\pi}\frac{\partial \chi_\nu{}^{\mu\sigma}}{\partial x^\sigma}, \tag{7}$$

where the Møller superpotential,

$$\chi_\nu{}^{\mu\sigma} = \sqrt{-g}\left(\frac{\partial g_{\nu\alpha}}{\partial x^\beta} - \frac{\partial g_{\nu\beta}}{\partial x^\alpha}\right) g^{\mu\beta} g^{\sigma\alpha}, \tag{8}$$

are quantities antisymmetric in the indices μ, σ. According to the definition of the Møller energy-momentum complex, the energy component is given as

$$\begin{aligned} E &= \int \Theta_0{}^0 dx^1 dx^2 dx^3 \\ &= \frac{1}{8\pi}\int \frac{\partial \chi_0{}^{0k}}{\partial x^k} dx^1 dx^2 dx^3, \end{aligned} \tag{9}$$

where the Latin index takes values from 1 to 3. However, in the case, the only nonvanishing component of Møller's superpotential is

$$\chi_0{}^{01} = 2\, m\, \gamma\, sin\theta. \tag{10}$$

Applying the Gauss theorem to (9) and using (10), we evaluate the integral over the surface of a sphere with radius r, and find the energy distribution is

$$E = m\,\gamma. \tag{11}$$

It is the same result as obtained by Virbhadra in the Weinberg prescription.

3 DISCUSSION

It is well-known that the subject of the energy-momentum localization is associated with much debate. In contradiction with Misner *et al.*[23], Cooperstock and Sarracino [24] gave their viewpoint that if the energy localization is meaningful for spherical system it is, also, meaningful for all systems. Also, Cooperstock [25] gave his opinion that the energy and momentum are confined to the regions of non-vanishing energy-momentum tensor of the matter and all non-gravitational fields. Bondi [26] sustained that a nonlocalizable form of energy is not admissible in relativity so its location can in principle be found.

We calculate the energy distribution of the gamma metric using the energy-momentum complex of Møller. The energy depends on the mass m. Thus, we get the same result as Virbhadra [21] obtained using the energy-momentum complex of Weinberg. This result sustains the opinion that different energy-momentum complexes could give the same expression for the energy distribution in a given space-time. As we noted, for some given space-times [17] the energy distribution according with the energy-momentum complex of Møller is the same as those calculated in the Einstein prescription. Our results sustain the conclusion of Lessner [27] that the Møller energy-momentum complex is an important concept of energy and momentum in general relativity. Also, the Møller energy-momentum complex allows to make the calculations in any coordinate system.

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